NOTES AND CORRESPONDENCE

Comments on "On an Improved Model for the Turbulent PBL"

Y. CHENG, V. M. CANUTO, AND A. M. HOWARD

NASA Goddard Institute for Space Studies, New York, New York

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1. The value of Ri_c

Kantha (2003, hereafter Kantha) begins his note commentary on our paper (Cheng et al. 2002, hereafter CCH) by recognizing that the CCH model is the first to have removed a major stumbling block in the application of Reynolds stress models (RSMs) to geophysical flows. These models, initiated in the 1970s (Mellor 1973; Mellor and Yamada 1974, 1982; called MY models) persistently predicted that turbulence would cease to exist at a value of the Richardson number Ri given by

$$\mathrm{Ri}_c = 0.19, \tag{1a}$$

while a large variety of data indicated that the true value is some 5 times larger:

$$\mathrm{Ri}_c \sim 1. \tag{1b}$$

It is usually stated that it was only in 1985 when it was realized (Martin 1985) that (1a) produced an unrealistically shallow ocean mixed layer (ML), whereas (1b) gave much more realistic ML depths. However, before Martin's important contribution and even before the MY models, Woods (1969) presented a very physical argument that led him to conclude in favor of (1b) rather than (1a). Regrettably, Woods' discussion, which is all the more important as it is not based on any specific model, is hardly ever cited in this context. Thus, before and after the MY models appeared, there were both theoretical (Woods 1969) and practical (Martin 1985) reasons to expect that any reliable mixing model should yield (1b) rather than (1a).

Twelve years after the 1982 MY paper, the failure by MY-like models to produce (1b) prompted Large et al. [1994; K-Profile Parameterization (KPP) model] to suspect that the turbulence-based models may be structurally incapable of producing the correct amount of mixing in stably stratified flows. Large et al. (1994) therefore adapted to the ocean case a heuristic mixing model previously developed by Troen and Mahrt (1986) for the atmosphere. While it is difficult to argue against the logic underlying the KPP model as of 1994, it seems fair to argue that if the CCH model and (1b) had existed in 1994, the arguments that motivated the KPP model would not have existed.

The main point of the CCH model is that it is not the RSM methodology per se that is unable to produce (1b); it is the quality of its physical content that determines the performance of the model.

2. The parameters λ_2, λ_3

As recognized by Kantha, the CCH model strived to include not only more complete pressure–velocity and pressure–temperature correlations, but also to eliminate the uncertainties regarding some of the constants by using results of the renormalization group (RNG; Canuto and Dubovikov 1996a,b; 1997a,b,c; 1998, 1999; Canuto et al. 1996, 1997a,b,c; 1999a,b) that bypassed and resolved the limitations of a previous attempt to apply RNG to turbulence (Yakhot and Orzsag 1986).

Regrettably, however, not all the constants could be determined from such an a priori method. Two of them, λ_2 and λ_3, govern the difference of the lateral and vertical components $\overline{v}^2$, $\overline{w}^2$ of the turbulent kinetic energy $e = (1/2)(\overline{u}^2 + \overline{v}^2 + \overline{w}^2)$, and their choice is the main subject of Kantha's note.

a. The data

Kantha claims that the data regarding $\overline{v}^2$ and $\overline{w}^2$ are not well sorted out as yet. Thus, one can choose, as CCH did,

$$\lambda_2 \neq \lambda_3, \qquad \overline{v}^2 \neq \overline{w}^2, \tag{2a}$$

or as Kantha suggests,

$$\lambda_2 = \lambda_3, \qquad \overline{v}^2 = w^2. \tag{2b}$$

Corresponding author address: Dr. V. M. Canuto, NASA GISS, 2880 Broadway, New York, NY 10025.
E-mail: vcanuto@giss.nasa.gov

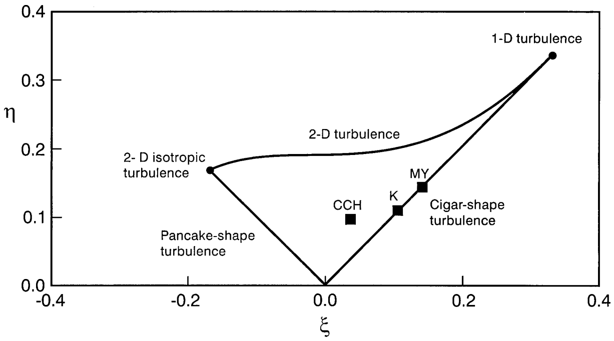

FIG. 1. The Lumley triangle with the CCH, Kantha's, and MY models.

From Mellor and Yamada's (1982) early statement "the fact that v and w are equal is not supported by the data," to the more recent work by Shih and Shabbir (1992), it follows that $\lambda_2 \neq \lambda_3$. In the planetary boundary layer (PBL) case, Fig. 9 of Moeng and Sullivan (1994) shows the sequence

$$\overline{w}^2 < \overline{v}^2 < \overline{u}^2, \tag{2c}$$

which confirms the Mellor–Yamada assertion that $v \neq w$. More recent data on *turbulence over hills*, specifically measurements from the Environmental Protection Agency (EPA) Russian Hill (RUSHIL) experiment give (Ying and Canuto 1996, 1997; Ying et al. 1994)

$$\overline{v}^2/e - 2/3 = 0.384, \tag{2d}$$

$$\overline{w}^2/e - 2/3 = -0.309. \tag{2e}$$

Kantha's choice, $\lambda_2 = \lambda_3$, cannot explain (2d) and (2e), whereas CCH's choice of $\lambda_{2,3}$ does.

b. Geometrical interpretation

Consider the dimensionless invariants II and III, which are defined as follows (Lumley and Newman 1977; Lumley 1978; Shih and Shabbir 1992):

$$-8e^2\mathrm{II} = b_{ij}b_{ij}, \qquad 24e^3\mathrm{III} = b_{ij}b_{jk}b_{ki}, \tag{2f}$$

where

$$b_{ij} = \overline{u_iu_j} - (2e/3)\delta_{ij}. \tag{2g}$$

Lumley and Newman (1977) and Lumley (1978) have devised what has become known as the "Lumley triangle" whose axes are

$$\xi \equiv \left(\frac{1}{2}\mathrm{III}\right)^{1/3}, \qquad \eta \equiv \left(-\frac{1}{3}\mathrm{II}\right)^{1/2}. \tag{2h}$$

The triangle is presented in Fig. 1. All turbulent flows are represented by points inside the triangle. Flows on the left-hand side of the triangle are axisymmetric, *pancake turbulence*:

$$\xi = -\eta. \tag{2i}$$

Flows on the right-hand side of the triangle are axisymmetric, *cigar-shaped turbulence*:

$$\xi = +\eta. \tag{2j}$$

Using the above expressions, we have computed ξ and η for the neutral case with $\lambda_2 \neq \lambda_3$ (CCH), $\lambda_2 = \lambda_3$ (Kantha and MY models). As one can see in Fig. 1, Kantha's model corresponds to having a cigar-shaped turbulence. However, this result is not consistent with the data by Moeng and Sullivan (1994) presented in Eq. (2c), which correspond to a shear driven turbulence with only slight stable stratification. On the other hand, the CCH model point in Fig. 1, while showing some tendency toward cigar-shaped turbulence, is away from the extreme $\xi = \eta$ case of axisymmetric turbulence, and thus seems more general. On these grounds, we feel that CCH's choice $\lambda_2 \neq \lambda_3$ is more appropriate than Kantha's $\lambda_2 = \lambda_3$.

3. Deardorff's limitation

To obtain the critical Richardson number Ri_c, CCH solved the equation that expresses production equals dissipation:

$$S_mG_m - S_hG_h - 2 = 0, \tag{3a}$$

where $G_m \equiv (\tau S)^2$ and $G_h \equiv (\tau N)^2$. The result is (1b). On the other hand, Kantha suggests that, in solving (3a) for Ri_c, one should account for the Deardorff limitation on the eddy length scale $\ell(\tfrac{1}{2}q^2 = e)$:

$$\ell < 0.53q/N, \quad \text{or} \tag{3b}$$

$$G_h(\max) = (0.53B_1)^2 \sim 100. \tag{3c}$$

If so, Eq. (3a) becomes

$$S_m\mathrm{Ri}_c^{-1}G_h^{\max} - S_hG_h^{\max} - 2 = 0, \tag{4a}$$

where

$$S_{m,h} \equiv S_{m,h}(\mathrm{Ri}_c^{-1}G_h^{\max}, G_h^{\max}), \tag{4b}$$

the solution of which is

$$\mathrm{Ri}_c = 0.52, \tag{4c}$$

instead of (1b). We now show that the adoption of (3c) in (3a) leads to incorrect results.

Let us assume (3c). The general form of the diffusivity (α may mean momentum, heat or salt; constants $= 1$; $\tau = 2e/\epsilon$) is

$$K_\alpha = e^2\epsilon^{-1}S_\alpha = \ell^2N(\tau N)^{-1}S_\alpha. \tag{4d}$$

Because Ri_c is defined as the point at which and beyond which turbulence disappears, the solution of the model must satisfy the relation

$$K_\alpha(\mathrm{Ri}_c) = 0. \tag{4e}$$

Because by (3c), τN is finite at Ri_c and because $S_\alpha(\mathrm{Ri}_c)$ does not vanish, the only way to satisfy (4e) is for l to vanish at $\mathrm{Ri}_c = 0.52$, which does not occur. Thus, one

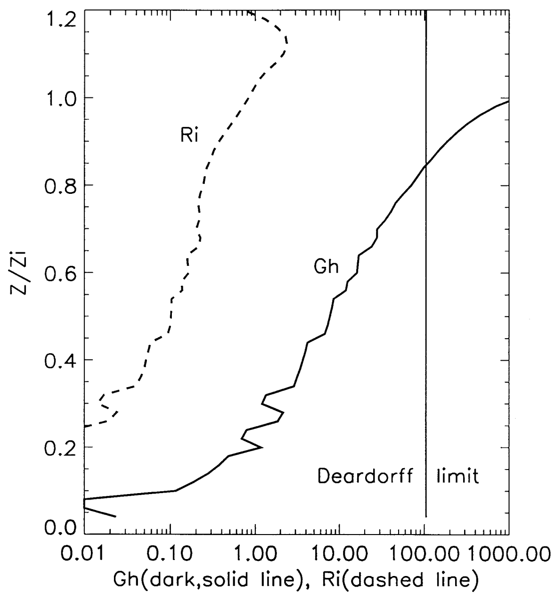

FIG. 2. The values of $G_h \equiv (\tau N)^2$ and of the Richardson number Ri vs height for the PBL. The results are obtained using LES data from Moeng and Sullivan (1993, personal communication). The Deardorff limitation is clearly valid for all Ri $<$ 0.25 but not for Ri $>$ 0.25.

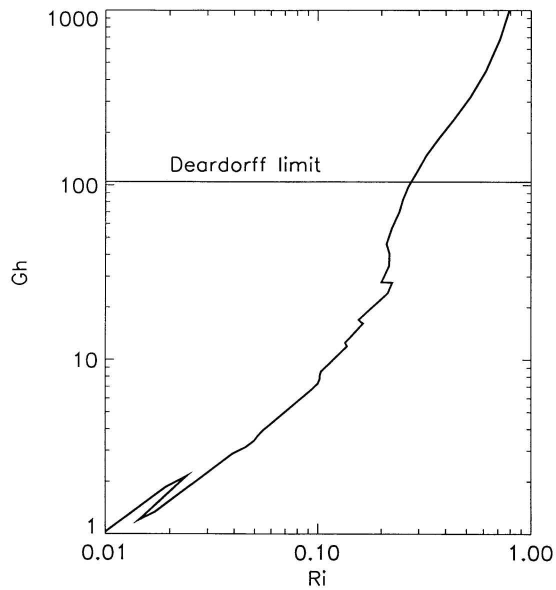

FIG. 3. The variable G_h vs Ri. This graph clearly exhibits the fact that for Ri $<$ 0.25, the Deardorff limitation applies. However, for large values of Ri, it is no longer valid.

concludes that (4e) is actually not satisfied; that is, turbulence is allowed to exist at a point where, by definition, it should be zero.

The second argument against the use of (3c) in solving (3a) is as follows. One can prove that *Deardorff's relation* (3c) *is valid only for small Ri, and therefore cannot be used near Ri* $\sim Ri_c$. A physical argument and published large eddy simulation (LES) data will prove the point. As Ri $\rightarrow$ Ri_c, turbulence gets increasingly weaker; that is, the nonlinear interactions become increasingly weaker and they no longer break up the large scales, which are the progenitors of the cascade process. Large scales are long lived, so much so that, in the limit of restoration of linearity (zero turbulence), their lifetime becomes very large ($\tau \rightarrow \infty$). *They are stable.* Since this is a model-independent argument, it should be a warning against applying the Deardorff limitation on τN in a region where it is known that $\tau \rightarrow \infty$. In turn, this leads to the suspicion that Deardorff's limitation must not be valid near Ri $\sim$ Ri_c but only for small Ri.

The second argument employs LES data kindly provided to us by Moeng and Sullivan. Only these LES data (no turbulence model) were used in constructing Figs. 2 and 3. In Fig. 2 we plot both Ri and G_h versus z. The values on the x axis are for both Ri and G_h. In Fig. 3 we plot directly G_h versus Ri. It is clear that for all Ri greater than

$$\mathrm{Ri} \geq 0.25, \tag{4g}$$

G_h (LES) becomes larger than Deardorff's limit (3c). One observes that Deardorff's limitation (3c) violates the LES data at high Ri and thus it should not be used there. In summary, use of (3c) in solving (3a) has the following consequences: (a) it gives a nonzero turbulence where, by definition, there is none; (b) at a more fundamental level, it is applied in a region where it fails to the maximum extent.

4. Nakanishi's relation

Kantha writes that the use of Nakanishi relation [Eq. (35) of CCH] is ''essential for the good agreement of the CCH results with the Kansas data.'' However, the key improvement brought about by CCH and shown in CCH's Figs. 7 and 8 occurs mostly in the *unstable region* where the Nakanishi relation was not used.

5. Conclusions

In conclusion, we have welcomed Kantha's comments, for they provided us an opportunity to clarify several important features of our work.

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REFERENCES

Canuto, V. M., and M. S. Dubovikov, 1996a: A dynamical model for turbulence: I. General formalism. *Phys. Fluids,* **8,** 571–586.

——, and ——, 1996b: A dynamical model for turbulence: II. Shear driven flows. *Phys. Fluids,* **8,** 587–598.

——, and ——, 1997a: A dynamical model for turbulence: V. The effect of rotation. *Phys. Fluids,* **9,** 2132–2140.

——, and ——, 1997b: Physical regimes and dimensional structure of rotating turbulence. *Phys. Rev. Lett.,* **78,** 666–669.

——, and ——, 1997c: A new approach to turbulence. *Int. J. Mod. Phys.,* **12,** 3121–3152.

——, and ——, 1998: Two scaling regimes for rotating Raleigh–Benard convection. *Phys. Rev. Lett.,* **80,** 281–284.

——, and ——, 1999: A dynamical model for turbulence: VII. The five invariants for shear driven flows. *Phys. Fluids,* **11,** 659–664.

——, ——, Y. Cheng, and A. Dienstfrey, 1996: A dynamical model for turbulence: III. Numerical results. *Phys. Fluids,* **8,** 599–613.

——, ——, and A. Dienstfrey, 1997a: A dynamical model for turbulence: IV. Buoyancy-driven flows. *Phys. Fluids,* **9,** 2118–2131.

——, ——, and ——, 1997b: Turbulent convection in a spectral model. *Phys. Rev. Lett.,* **78,** 662–665.

——, ——, and D. J. Wielaard 1997c: A dynamical model for turbulence: VI. Two dimensional turbulence. *Phys. Fluids,* **9,** 2141–2147.

——, ——, and G. Yu, 1999a: A dynamical model for turbulence: VIII. IR and UV Reynolds stress spectra for shear driven flows. *Phys. Fluids,* **11,** 656–677.

——, ——, and ——, 1999b: A dynamical model for turbulence: IX. The Reynolds stress for shear driven flows. *Phys. Fluids,* **11,** 678–694.

Cheng, Y., V. M. Canuto, and A. M. Howard, 2002: An improved model for the turbulent PBL. *J. Atmos. Sci.,* **59,** 1550–1565.

Kantha, L., 2003: On an improved model for the turbulent PBL. *J. Atmos. Sci.,* **60,** 2239–2246.

Large, W. G., J. C. McWilliams, and C. C. Doney, 1994: Oceanic vertical mixing. *Rev. Geophys.,* **32,** 363–403.

Lumley, J. L., 1978: Computational modeling of turbulent flows. *Advances in Applied Mechanics,* Vol. 18, Academic Press, 123–175.

——, and G. R. Newman, 1977: The return to isotropy of homogeneous turbulence. *J. Fluid Mech.,* **82,** 161–178.

Martin, P., 1985: Simulation of the mixed layer at OWS November and Papa with several models. *J. Geophys. Res.,* **90,** 903–916.

Mellor, G. L., 1973: Analytic prediction of the properties of stratified planetary surface layers. *J. Atmos. Sci.,* **30,** 1061–1069.

——, and T. Yamada, 1974: A hierarchy of turbulent closure models for planetary boundary layers. *J. Atmos. Sci.,* **31,** 1791–1806.

——, and ——, 1982: Development of a turbulence closure model for geophysical fluid problems. *Rev. Geophys. Space Phys.,* **20,** 851–875.

Moeng, C.-H., and P. P. Sullivan, 1994: A comparison of shear and buoyancy driven planetary boundary layer, *J. Atmos. Sci.,* **51,** 999–1022.

Shih, T. S., and A. Shabbir, 1992: Advances in modeling the pressure correlation terms in the second moment equations. *Studies in Turbulence,* T. B. Gatski, S. Sarkar, and C. G. Speziale, Eds., Springer-Verlag, 91–115.

Troen, I. B., and L. Mahrt, 1986: A simple model of the atmospheric boundary layer: Sensitivity to surface evaporation. *Bound.-Layer Meteor.,* **37,** 129–148.

Woods, J. D., 1969: On Richardson's number as a criterion for laminar–turbulent–laminar transition in the ocean and atmosphere. *Radio Sci.,* **4,** 1289–1298.

Yakhot, V., and S. A. Orszag, 1986: Renormalization group analysis of turbulence, I. Basic theory. *J. Comput. Phys.,* **1,** 3–85.

Ying, R., and V. M. Canuto, 1996: Turbulence modeling over over two-dimensional hills using an algebraic Reynolds Stress expression. *Bound.-Layer Meteor.,* **77,** 69–99.

——, and ——, 1997: Numerical simulation of flow over two-dimensional hills using a second-order turbulence closure model. *Bound.-Layer Meteor.,* **85,** 447–474.

——, ——, and R. M. Ypma, 1994: Numerical simulation of flow data over two-dimensional hills. *Bound.-Layer Meteor.,* **70,** 401–427.